DYNAMIC STRIPES, RVB SPIN LIQUID AND HIGH T_C SUPERCONDUCTIVITY — A GAME OF TWO PLAYERS

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1. INTRODUCTION

In this paper we will explore the interplay between RVB spin liquid and 1D stripes in search of novel physics which might be relevant to high Tc cuprates. Two scenarios are discussed in this context, namely how RVB spin liquid background affects stripe dynamics and visa versa. We then relate the theoretical results to experiments on cuprates.

2. REVIEW OF RVB SPIN LIQUID

The brilliant idea of RVB states was advanced by Anderson soon after the discovery of high T_c superconductivity [1]. It was proposed to offer a plausible answer to the following essential question posed by cuprate superconductors, "how is an antiferromagnetic insulator driven toward a superconductor by increasing mobile charge carriers?". The ground states of undoped cuprates have Neel orders. By introducing carriers, the kinetic energy of the carriers starts competing with the exchange energy, the source of Neel order. This competition leads to an energetically favored RVB state. The idea of the RVB state is based on singlet bonds (valence bonds) where spin singlets are formed by each pair of spin 1/2 particles. The spin singlet bond optimize the exchange energy between the two sites associated with the bond. However, since each spin can only form a singlet with one of the spins of its neighbors, the energy cost of a fixed configuration is too high. To reduce the energy cost, Anderson introduced resonance processes between the different configurations, which allows a superposition of singlet bonds with different neighbours. This leads to the concept of the RVB state which is simply a coherent superposition of different configurations of singlet pairs.

The RVB state can be viewed as a spin liquid without symmetry broken, and thus, does not have long range order. Since the RVB state was proposed for two dimension Hubbard model which can be reduced to t-J model in large U limit, it is rather remarkable that Anderson and coworkers [2–4] showed that it supports spin-charge separation, which was only known to be a general feature in strongly correlated 1-D systems. The low energy excitations in the RVB vacuum are represented by two separated quasi-particles, spinon which has spin $\frac{1}{2}$ and charge zero, and holon which has zero spin but charge one. In this picture, a hole doped to the RVB state undergoes spin-charge separation. The charge degree of freedom is able to move freely in the spin-liquid and becomes superconducting upon condensation, which naturally provides a scenario for the transition from the insulating phase to superconducting phase.

RVB seems to agree with the anomalous normal state properties of cuprates observed in many experiments. The most impressive explanation is the broad quasiparticle spectra seen in the normal states of most underdoped and optimally doped materials. Moreover, the prominent d-wave order parameter of high Tc cuprates is naturally accommodated in the context of RVB theory [5, 6], as a simple solution of the mean field Hamiltonian containing nearest-neighbor antiferromagnetic exchange although the Anderson's original hypothesis was extended S wave. To see this, we consider the following simple Hamiltonian,

$$\begin{aligned} H_{RVB} &= -J \sum_{<ij>} b_{ij}^{+} b_{ij} \qquad (1) \\ &= -J \sum_{<ij>} (\Delta_{ij}^{*} b_{ij} + \Delta_{ij} b_{ij}^{+} - |\Delta_{ij}|^2), \end{aligned}$$

where $b_{ij}^{+} = \frac{1}{\sqrt{2}}[c_{i,\uparrow}^{+} c_{j,\downarrow}^{+} - c_{i,\downarrow}^{+} c_{j,\uparrow}^{+}]$, Through a routine Hartree-Fork decoupling, we introduce the RVB order parameter, Δ_{ij}, which is the mean field average of b_{ij} defined on each bond between 2 nearest neighbors. Then we obtain the RVB mean field Hamiltonian in the momentum space,

$$H_{RVB} = \frac{-J}{\sqrt{2V}} \sum_{\mathbf{k},\mathbf{q}} [\Delta^*_{\mathbf{k},\mathbf{q}} (c_{\mathbf{q},\downarrow} c_{\mathbf{k}-\mathbf{q},\uparrow} - c_{\mathbf{q},\uparrow} c_{\mathbf{k}-\mathbf{q},\downarrow}) + h.c.], \tag{2}$$

where $\Delta_{\mathbf{k},\mathbf{q}} = \sum_{\hat{\delta}} \Delta_{\mathbf{k},\hat{\delta}} e^{i\mathbf{q}\hat{\delta}}$, and $\Delta_{\mathbf{k},\hat{\delta}} = \frac{1}{\sqrt{V}} \sum_{\mathbf{r_i}} \Delta_{i,i+\hat{\delta}} e^{-i\mathbf{k}\mathbf{r_i}}$ ($\hat{\delta} = \pm\hat{\mathbf{x}}, \pm\hat{\mathbf{y}}$). The 'd-wave' solution with $\Delta^d_k = J(\cos k_x - \cos k_y)$ can be naturally introduced in the above formulations although there are several different solutions at half-filling, such as extended S wave, $s + id$ wave and flux phase.

Unfortunately, the original RVB solutions of the Heisenberg Hamiltonian turned out to have higher energy than the conventional Neel state with antiferromagnetic long range order at half filling. Therefore attentions had been focused on what happen at finite doping where t-J model is thought to be a good model. However the above RVB solutions cannot be easily generalized to the doped case, for example one attempt to modify the 'flux phase' by changing the flux from π resulted in the breaking down of time-reversal invariance which contradict the experiments. In a recent paper, Anderson revived the RVB idea by suggesting a generalization of the 's+id' or flux phase that preserves the time-reversal symmetry [7]. Another way of introducing RVB state in finite doping is by the charge in-homogeneity which has been established recently and will be the focus of this paper.

RVB scenarios had later evolved into several paths of theories. The short range RVB [8], in which the valence bonds only exist between the nearest neighbor sites, is the closest model to Anderson's original vision. The simple quantum dimer model was developed to capture the short range RVB physics [9] However, the quantum dimer model failed to produce a spin liquid phase on the square lattice. Recently progress has been made in triangle lattice where RVB phase can be constructed [10]. The long range RVB has been treated in different Gauge theories. Based on $U(1)$ and $SU(2)$, the t-J model has been studied extensively close to the RVB type of physics [11]. Recently, Z_2 gauge theory [12] provides a precise description of the deconfined phases in $2 + 1$ dimensions, namely, spin-charge separation which is the essential physics of the original RVB. The flux phase version has also been emphasized in other types of theories where the time reversal symmetry is broken, including anyon superconductivity [13] and D-density wave [14].

Recently Anderson also proposed a revision of his RVB theory to bring it in agreement with modern simulations and experimental results: in addition to the original non-Fermi liquid with full Fermi surface and separated spin and charge degrees of freedom (the Luttinger liquid), at finite hole doping, there exists another non-Fermi liquid phase with the presence of RVB spin gap and spinons at gap nodes (nodons). Superconductivity can emerge from both NFL phases and may be described by two distinct mechanisms [7].

Lately there has been renewed interest in the plausible relevance of RVB correlation to cuprate physics at relatively high energy scale, motivated both experimentally and theoretically. Recent ARPES result on $Ca_2CuO_2Cl_2$ by F. Ronning et al. [15] reveals the presence of a d-wave dispersion along the remnant Fermi surface and a Dirac like dispersion isotropically focused around $(\pi/2, \pi/2)$, which is exactly what was predicted for the "π-flux phase" [5,6] of RVB spin liquid, where $\epsilon(k) = J\sqrt{\cos^2 k_x + \cos^2 k_y}$, and can not be described within the spin density wave picture although the latter can account for the low-lying spin excitations in the Neel state. Further numerical results also support the presence of a local RVB spin liquid state around a doped hole with momentum $\mathbf{k} = (\pi, \mathbf{0})$ [16], accompanied by an anti-phase of spins around the hole which may be relevant to the generation of anti-phase domain walls in striped phase of cuprates. Theoretically, Kim and Lee show that Neel order can be restored in π-flux phase through dynamical mass generation of gauge fluctuations at low temperature [17] , which points toward an emerging consistent RVB picture spanning from ground state to high energy scale physics [18].

3. REVIEW OF STRIPE PHASE

As mentioned in above section, the RVB state is a homogeneous state without any broken symmetries. However, mean field calculations in theoretical models [19–21] suggested possible inhomogeneous states. In these states, the charge carriers have a tendency to self-organize into one dimensional objects called stripes. The stripe orders are characterized by spontaneously broken translational symmetries. In doped antiferromagnets, stripes can be viewed as charge-rivers formed along spontaneously generated domain walls between antiferromagnetic insulating regions.

When holes are doped into antiferromagnetic insulators, their motion frustrates the antiferromagnetic background. To save the Heisenberg exchange energy, holes should be localized but localized holes cost large kinetic energy. As a result, if holes were neutral, they would tend to aggregate together which results in phase separation when doping level is low, i.e. there is an effective short range attraction. In the hole poor region, the exchange energy is saved while in the hole rich region the kinetic energy is saved. However, the long range Coulomb repulsion among holes is against this tendency. Compromise between these interactions at short and long length scales leads to the configuration of one dimensional stripes along which holes can move freely. Along the direction perpendicular to stripes, holes are confined thus the zero point kinetic energy causes the transverse oscillation of stripes. The spacing between stripes is determined by the Coulomb energy and the competition among the three energies determines the filling in one stripe. Theoretically, antiferromagnetic orders across stripes may be antiphase or in phase [22–25] depending on the filling in stripes and whether stripes are bond-centered or site-centered, while in the experiment only the antiphase stripes are observed. Experimentally, doping caused charge inhomogenity is widely observed: solitons in 1D, linear 'rivers of charge' in 2D, and planes of charge in 3D, as exemplified by organic conductors, cuprates or

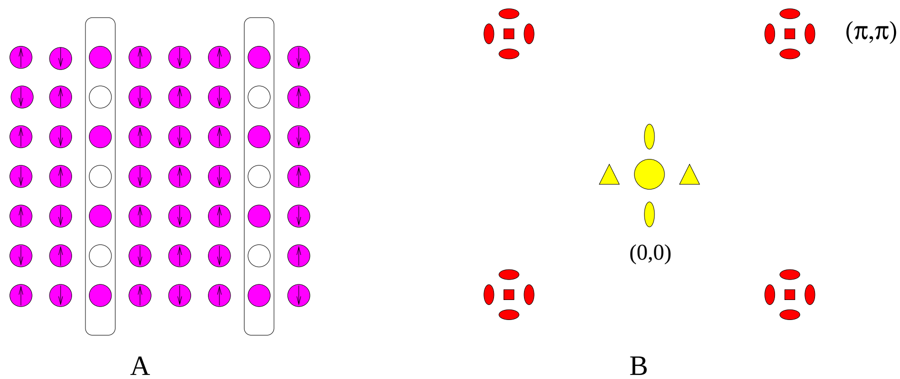

Figure 1: A. Stripe configuration with 1/8 filling. Holes are confined in stripes. The charge period is 4 and the antiferromagtic order period is 8 with a π phase shift across stripes. B. Illustration of antiferromagnetic Bragg peaks around (π, π) and fundmental lattice peaks around (0,0). Upon doping, commensurate peaks are shifted to incommensurate peaks. For antiferromagnetic peaks, $(\pi, \pi) \longrightarrow (\pi \pm 2\pi x/a\ \vec{n})$ and $(0,0) \longrightarrow \pi \pm 4\pi x/a\ \vec{n}$.

nickelates, and manganites respectively. They have the common nature of "topological defects". Even in quantum Hall systems, the stripe phases are believed to have been observed [26, 27]. Below we will review both the experimental evidence of stripes and some theoretical work.

Neutron scattering is one of the most powerful tools to investigate stripes. Usually neutrons can only detect directly the spin configuration instead of the charge distribution. However, neutron scattering can tell us the period of distortion of ionic displacements induced by the charge modulation and give the charge information indirectly. In $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$(LNSCO) systems, static stripes were detected by Tranquada *et al* [28] where the replacement of some La atoms to Nd is to distort the lattice to pin down the stripe order. They found the magnetic Bragg peak is shifted from $Q = (\pi, \pi)$ to the incommensurate peaks $Q \pm 2\pi x/a\ \vec{n}$ and the fundamental lattice Bragg peak is shifted to $4x\pi/a\ \vec{n}$, where x is the doping level, a the lattice constant and $\vec{n}$ is along Cu-O bond directions. These features can be explained as follows: The period of charge strips is $a/2x$ and that of spin antiferromagnetic configurations is doubled as a/x which are locally commensurate but have π phase shifts across charge stripes. The charge order has also been confirmed by high energy x-ray diffraction [31]. In this material, charge order formation occurs at higher temperature than spin ordering. Stripes are found not only in normal state but also to coexist with superconductivity in the superconducting state. The Bragg peaks are observed in both x and y directions which can be explained as that the orientations of stripes in neighboring layers are perpendicular to each other.

Elastic neutron scattering in $La_{2-x}Sr_xCuO_4$ systems found static stripes from doping $x = 0.02 \sim 0.13$. From $x = 0.02 \sim 0.05$, where samples are in normal states even at zero temperature, stripes are along the diagonal direction in the CuO plane [29]. From $x = 0.05 \sim 0.13$, stripes change orientation to be vertical and samples enter the superconducting state at low temperatures [30]. Inelastic neutron scattering in this system also detected incommensurate magnetic peaks at doping level from $x = 0.13 \sim 0.25$ which can be interpreted as slowly fluctuating stripes.

In other families of high T_c superconductors such as $YBaCu_2O_{7-\delta}$ (YBCO) and $Bi_2Sr_2CaCu_2O_{8+\delta}$(BSCCO), it is hard to see static stripes. A recent neutron scattering experiment by Mook *et al* claimed the observation of static stripes in $YBaCu_2O_{6.35}$ [32]. More commonly, incommensurate antiferromagnetic spin fluctuation are found in underdoped and optimally doped YBCO systems [33–36]. A weak signal of incommensurate spin fluctuation is also observed in BSCCO samples [37] although the neutron experiments are difficult to perform in BSCCO because of weak coupling between layers and lack of large crystal of BSCCO suitable for neutron scattering. The incommensurability observed in YBCO and LSCO is consistent with the dynamic fluctuating stripe picture at least in underdoped region.

Scanning Tunneling Microscope(STM) is an important experiment method to investigate electronic spectra directly with local resolutions. The requirement for high surface quality makes BSCCO most suitable candidate. Recently, the local stripe-like static modulation of density was observed in nearly optimally doped $BSCCO$ [38]. This modulation is aligned with the Cu-O bonds, with a periodicity of four lattice constants. It is not clear whether the modulation is really a one-dimensional structure. Nevertheless, the periodicity is consistent with the prediction from stripe ordering. In the future, STM certainly will provide more useful information to clarify this issue.

Angular resolved photoemission spectroscopy(ARPES) experiments were also carried out on LNSCO and LSCO systems [39,40]. After integration over energy, spectral weight was showed to confine within the the straight segments $k_x = \pm\pi/4$ and $k_y = \pm\pi/4$ near the antinode regions $(\pi, 0)$ and $(0, \pi)$. Low energy excitations near the node regions are strongly suppressed, but there is still some spectra weight along nodal direction remaining. The feature of flat band near Brillouin zorn boundary can be easily understood in the picture of static stripes. However, the existence of nodal particles is a challenging problem. Granath *et al* [41] showed that it is still possible to have d-wave nodal particles in the stripe phase by considering the coupling between stripes and their environment. There is still much work to do to recouncil the nodal particles with the stripe picture.

The transport measurements including in-plane resistivity and magnetoresistence(MR), Hall coefficient also provide important evidence for stripes. The in-plane resistivity $\rho_a b(T)$ measurement on YBCO [42] and LSCO [43] showed that $\rho_{ab}(T)$ remains metallic behavior across the Neel temperature (T_N) and the expected insulating behavior only appears at the temperature much lower than T_N. One nature explanation is that holes are concentrated in stripes below T_N, thus metallic behavior of $\rho_{ab}(T)$ does not change much until couplings among stripes, such as charge density wave coupling, bring insulating states at much lower temperatures. MR measurements $\delta\rho_{ab}/\rho_{ab}$ in YBCO [42] showed an interesting d-wave like behavior with the direction of magnetic field H respective to the current I, i.e. $\delta\rho_{ab}/\rho_{ab}$ is negative or positive when H is longitudinal or transverse with I. It was argued that magnetic field H can align stripes toward the field directions. The in-plane resistivity anisotropy in untwinned single crystals of $La_{2x}Sr_xCuO_4(x = 0.020.04)$ and $YBa_2Cu_3O_x(x = 6.357.0)$ [44] showed strong temperature and doping dependence in the lightly hole-doped region, which can not be explained by crystalline anisotropy alone. The Hall coefficient and doping dependence of resistivity in LNSCO [45]. When the doping density $x < 1/8$, the low temperature Hall coefficient is largely suppressed while it is still large when $x > 1/8$. This can be understood as that stripes are formed at low temperature when $x < 1/8$ and charge transport has the one-dimensional feature.

In short, the presence of stripe formulation is a general phenomenon in high Tc cuprates. Various experimental results can be understood in a unified picture of stripe phase. Quarter-filled [46] metallic stripes begin to form when doping level $x > 0.06$ with antiferromagnetic insulating regions intervening among them. The inter-stripe distance is proportional to $1/x$. When $x > 1/8$, inter-stripe distance saturates while incommensurate stripe configurations remain. Extra holes can overflow into the insulating regions, i.e. stripes oscillate strongly in transverse direction. At last, when transverse oscillation is strong enough, the stripe order ceases to exist and translational symmetry is restored.

Although there are overwhelming experimental evidences for stripes, many open questions remain to be answered [47]. The most important one is the relation of stripes with superconductivity: Are stripes helpful or harmful to superconductivity? Emery*et al* [48] gave a scenario called “spin gap proximity effect” based on stripes interacting with active environments. In this formulation, at a high temperature scale T_1^* the charge inhomogeneity i.e. stripes happens and also local antiferromagetic correlation develops in insulating regions. Stripes are one dimensional electron gases (1DEG). For the 1DEG analog of superconductivity, the amplitude of pairing order is determined by spin sector, i.e. the spin gap, and the phase of pair comes from the charge sector. However, for a pure 1DEG with positive repulsion, the pairing correlation is irrelevant, i.e. there is no spin gap. In the presence of pair tunneling, if there exists a spin gap in the environment, it may induce a spin gap in the stripe, i.e. the formation of local paring. This happens at a lower temperature scale T_2^*. A lower temperature scale T_c is needed for global phase coherence among all the stripes by Josephosn coupling i.e. a dimension cross over to fix the superconductivity long range order. However, couplings among stripes usually bring charge density wave (CDW) order instead of superconducting order if the interaction is repulsive because CDW correlation is more relevant that pairing correlation. Kivelson *et al* [49] showed that strong transverse oscillation of stripes can suppress CDW coherence and enhance paring coupling, i.e. although static stripes is harmful to superconductivity, dynamic stripes may even be helpful to it. They further formulated the concept of electronic liuqid crystal: Stripes can be viewed as smectic phase where the translation symmetry along one direction is broken. It can be viewed as intermediate state between crystal phase where the translation symmetry along two directions are broken and nematic phase where the translation symmetry is restored but the rotation symmetry is broken. When all the translational and rotation symmetries are restored, it is the isotropic or disordered phase. Superconductivity may happen in smectic, nematic, and isotropic phases. There also exists a strong empirical evidence, the Yamada plot [30], to support the relation between stripe and superconductivity, i.e. T_c is linear with incommensurability seen in neutron scattering which is the inverse spacing between stripes in underdoped cuprates.

There are also several other theories about the mechanism of superconductivity within the picture of stripes. Holder et al. [50] considered the effect of stripe formation on the superconcutivity using an anharmonic spin-charge-phononn interaction model. Neto [51] modeled the hole rich stripe as Luttinger liquid, and the hole poor region as two dimensional interacting pair gas to study the formation of supercoundcutivity. Stripe piciture is a promising mechnism for high Tc superconducivity but there is still a long way to go to establish a broadly accepted comprehensive framework.

4. HOW RVB SPIN LIQUID BACKGROUND AFFECTS STRIPE DYNAMICS

Here let's address the following important issue : assuming the presence of preformed stripes coupled to a half-filling spin liquid background, can we improve our understanding of the intriguing physics revealed in both normal state and superconducting state of cuprates? Such exploration has been recently attempted in [48, 52], and some interesting results have been reported . Most of these attempts treated the stripe as a 1D or quasi-1D Luttinger Liquid coupled to other stripes perpendicular to it [53], or coupled to insulating background which is either modeled as a classical antiferromagnetic insulator [54] or as another 1D Luttinger Liquid [48]. In contrast, I emphasize the importance of coupling a stripe as a Luttinger Liquid to a truly anomalous 2D insulating background, which has its hidden unconventional nature of RVB (Resonating Valence Bond) spin liquid, under the classical apparel of antiferromagnetic (AFM) order. It is expected that by coupling the above two higly unconventional quantum systems, new physics may arise which is potentially relevant to the understanding of high Tc cuprates.

Here is a brief summary of the results:

Through single particle hopping between 1D stripes and 2D RVB background, a normal state pseudo-gap Δ_n is "induced" inside the stripe's spin sector, which coincides with the mechanism of spin gap formation in a class of 1D electron systems named after Luther and Emery [55] . Further more, inside the superconducting state, the presence of global phase coherence in RVB order parameter contributes to an even more relevant pairing coupling to 1D stripe, which results in a gap of Δ_{sc} opening in both spin and charge sectors. Experimental consequences of 2 quantitatively different gaps are discussed, and good agreement is found with ARPES results [56]. I also discuss the Josephson coupling E_J between neighboring stripes that determines the energy scale of T_c, and the evolution of single particle spectral line-shape with doping.

As reviewed in Section one, The RVB state is described by a coherent superstition of different configurations of valence bonds, which was expected to be a reasonable approximation to the ground state of insulating spin 1/2 Heisenberg Model when Neel order is suppressed by frustration or hole doping.

lls in striped phase of cuprates. Theoretically, Kim and Lee show that Neel order can be restored in π-flux phase through dynamical mass generation of gauge fluctuations at low temperature [17] , which points toward an emerging consistent RVB picture spanning from ground state to high energy scale physics [18].

Here I suggest that one can model the environment of a quarter-filled stripe as a RVB spin liquid, which is coupled with the stripe by a single particle hopping term that conserves the momentum along the stripe direction [57]. To get started, one can first ignore the inter-stripe correlation in the normal state of underdoped cuprates, considering the strong incoherence revealed by experiments. The total Hamiltonian is given by

$$\begin{aligned} H(c,c^+,d,d^+) &= H_{1D}(d,d^+) + H_{RVB}(c,c^+) \\ &+ H_{couple}(c,c^+,d,d^+), \end{aligned} \tag{3}$$

where c, c^+ and d, d^+ represent the annihilation and creation operators of a single particle in 2D RVB background and 1D stripe, respectively.

$$H_{couple}(c,c^+,d,d^+) = \sum_{\mathbf{k},q,\sigma} V c^+_{\mathbf{k},\sigma} d_{q,\sigma} \delta_{k_x,q} + h.c.$$

where only horizontal stripe is considered, $\mathbf{k} = (k_x, k_y)$, and momentum conservation is ensured by requiring $k_x = q$. V gives the hopping matrix element, which is crutial in deciding different energy scales relevant to cuprates , as will be discussed later [57].

A routine Hartree-Fork decoupling is applied to the RVB Hamiltonian H_{RVB} [1],

$$\begin{aligned} H_{RVB} &= -J \sum_{<ij>} b^+_{ij} b_{ij} \\ &= -J \sum_{<ij>} (\Delta^*_{ij} b_{ij} + \Delta_{ij} b^+_{ij} - |\Delta_{ij}|^2), \end{aligned} \tag{4}$$

where $b^+_{ij} = \frac{1}{\sqrt{2}}[c^+_{i,\uparrow} c^+_{j,\downarrow} - c^+_{i,\downarrow} c^+_{j,\uparrow}]$, Δ_{ij} is the RVB order parameter defined on each bond between 2 nearest neighbors, which is reduced to the mean field average of b_{ij} operator at the saddle point level. Then one can change into the momentum space, that is

$$H_{RVB} = \frac{-J}{\sqrt{2V}} \sum_{\mathbf{k},\mathbf{q}} [\Delta^*_{\mathbf{k},\mathbf{q}} (c_{\mathbf{q},\downarrow} c_{\mathbf{k}-\mathbf{q},\uparrow} - c_{\mathbf{q},\uparrow} c_{\mathbf{k}-\mathbf{q},\downarrow}) + h.c.], \tag{5}$$

where $\Delta_{\mathbf{k},\mathbf{q}} = \sum_{\hat{\delta}} \Delta_{\mathbf{k},\hat{\delta}} e^{i\mathbf{q}\hat{\delta}}$, and $\Delta_{\mathbf{k},\hat{\delta}} = \frac{1}{\sqrt{V}} \sum_{\mathbf{r_i}} \Delta_{i,i+\hat{\delta}} e^{-i\mathbf{k}\mathbf{r_i}}$ ($\hat{\delta} = \pm\hat{\mathbf{x}}, \pm\hat{\mathbf{y}}$).

There are many possible mean field states in RVB theory [58], among which the "π-flux phase" is selected here, because of its low energy, conservation of time-reversal symmetry and possible connection to AFM long range order [17]. In "π-flux phase", $\Delta_{ij} = \Delta_0 e^{i\phi_{ij}}$ is chosen to have uniform amplitude Δ_0, while its phase ϕ_{ij} is selected to ensure that staggered $+\pi$ and $-\pi$ flux is threaded through each plaquette. For convenience, one can choose $\phi_{ij} = \pm\pi/4$. Therefore, H_{RVB} is simplified to

$$\begin{aligned} H_{RVB} &= -J \sum_{\mathbf{q}} [\Delta_0^* \gamma(\mathbf{q}) (c_{\mathbf{q},\downarrow} c_{-\mathbf{q},\uparrow} - c_{\mathbf{q},\uparrow} c_{-\mathbf{q},\downarrow}) + h.c.] \\ &+ iJ \sum_{\mathbf{q}} [\Delta_0^* \eta(\mathbf{q}) (c_{\mathbf{q},\downarrow} c_{\hat{\pi}-\mathbf{q},\uparrow} - c_{\mathbf{q},\uparrow} c_{\hat{\pi}-\mathbf{q},\downarrow}) - h.c.], \end{aligned} \tag{6}$$

where $\gamma(\mathbf{q}) = \cos q_x + \cos q_y$, $\eta(\mathbf{q}) = \cos q_x - \cos q_y$ and $\hat{\pi} = (\pi, \pi)$. Then we perform Euclidean path integral over the 2D degrees of freedom and obtain the low energy effective action for 1D stripe as follows

$$\begin{aligned} e^{-S_{eff}} &= \exp\{-\int_0^\beta H_{eff}(d, d^+) d\tau + \frac{1}{\beta} \sum_n i\omega_n d^+ d\} \\ &= \int dc dc^+ \exp\{-\int_0^\beta [H_{1D}(d, d^+) + H_{couple}(c, c^+, d, d^+) \\ &+ H_{RVB}(c, c^+)] d\tau + \frac{1}{\beta} \sum_n i\omega_n (c^+ c + d^+ d)\}, \end{aligned} \tag{7}$$

where $\omega_n = \frac{\pi n}{\beta}$ (n is odd integer), $\beta = \frac{1}{k_B T}$.

Assuming $J|\Delta_0| >> V$, one can change back to 1D coordinate system and get

$$\begin{aligned} H_{eff} &= H_{1D}(d, d^+) - \frac{V^2}{16J} \sum_l [\frac{1}{\Delta_0^*} (d_{l,\downarrow} d_{l+1,\uparrow} - d_{l,\uparrow} d_{l+1,\downarrow}) \\ &+ h.c.] - i\frac{V^2}{16J} \sum_l [\frac{(-1)^l}{\Delta_0^*} (d_{l,\downarrow} d_{l+1,\uparrow} - d_{l,\uparrow} d_{l+1,\downarrow}) - h.c.]. \end{aligned} \tag{8}$$

Then go to the continuum limit, $d_{l,\sigma} \to \sqrt{a}\Psi(x = la)_\sigma$, with the size of unit cell $a \to 0$ and retain only the slow varying part of H_{eff}, we finally arrive at the following correction to H_{1D} due to its coupling to RVB background,

$$\Delta H_{eff} = g \int dx \Psi_\downarrow(x) \Psi_\uparrow(x) + h.c., \tag{9}$$

where $g = -\frac{V^2 \cos k_F}{8J\Delta_0^*}$, and $k_F = \pi/4$ for quarter-filled stripe. We note that the 1D anomalous propagator is induced in stripes through hopping V by the strong pairing correlation inherent to the RVB background. This mechanism is central to the pairing process among mobile carriers inside stripes, via which superconductivity becomes viable.

Based on the above result, I will discuss both normal state and superconducting state , respectively. Let us first address the issue of normal state pseudo-gap , which is deemed as very important but remains highly controversial. To treat normal state here, one can take the strong phase fluctuation in RVB order parameter into account , while its amplitude is basically non-zero and much less fluctuating. Therefore, one can integrate out the phase of Δ_0, and get

$$H_{eff} = H_{1D}(d, d^+) + g_1 \int dx \Psi_\uparrow^+ \Psi_\uparrow \Psi_\downarrow^+ \Psi_\downarrow,$$

where $g_1 = \frac{-g^2 a}{2}$, which can be treated with the standard bosonization technique [59]as follows

$$\begin{aligned} H_{eff} &= \int dx \{[(\frac{K_c u_c}{2} - \frac{g_1}{2\pi})\Pi_c^2 + \frac{u_c}{2K_c}(\partial_x \Phi_c)^2] \\ &+ [(\frac{u_s}{2} + \frac{g_1}{2\pi})\Pi_s^2 + (\frac{u_s}{2} + \frac{g_1}{2\pi})(\partial_x \Phi_s)^2] \\ &+ g_1 \cos(\sqrt{8\pi}\Phi_s)\}, \end{aligned} \tag{10}$$

where Φ_c, Π_c, and Φ_s, Π_s , are conjugated boson operators representing density fluctuations in charge and spin sectors of 1D Luttinger Liquid [59], respectively. u_c, u_s are the corresponding propagating velocities, and K_c is a parameter of interaction.

In terms of renormalization group formulation, $g_1 \cos(\sqrt{8\pi}\Phi_s)$ in H_{eff} is marginally relevant, which results in the opening of a spectral gap in spin sector (spin gap)

$$\Delta_s \propto \sqrt{|g_1|} \exp{(\frac{v}{2\pi g_1})},$$

which is here identified as the normal state pseudo-gap Δ_n that leads to spectral weight depletion in low energy spin fluctuations and single particle spectrum, while the charge excitations remain gapless, which give rise to metallic transporting along the stripe. This is of the same principle as the early results by Luther and Emery in exploration of spin gap formation as an instability of Luttinger Liquid [55]. Further more, the effect of g_1 on charge sector is also physically important, it leads to $K_c > 1$, which ensures that singlet superconducting fluctuation dominates over CDW (charge density wave) correlation, and drives the system close to the opening of charge gap and superconducting phase transition that accompanies it (as will be clarified later).

Now let's turn to the superconducting state. It is generally agreed that global phase coherence is established at $T < T_c$, so that strong phase fluctuation in $\Delta_{ij} = \Delta_0 e^{i\phi_{ij}}$ is quenched, and the relevant correction to H_{1D} becomes Eq[9] itself with Δ_0 replaced by its average magnitude.

Then standard bosonization gives

$$\Delta H_{eff} = 2gd_u \int \cos(\sqrt{2\pi}\Theta_c)\sin(\sqrt{2\pi}\Phi_s)dx,$$

The scaling dimension of ΔH_{eff} is $\frac{1}{2}+\frac{1}{2K_c}$, so it is generally relevant except for very strong repulsive interactions(i.e. $K_c < 1/3$). Unlike the normal state case discussed before, in ΔH_{eff} both spin sector and charge sector are coupled together by a relevant effective interaction and spin-charge separation typical of a Luttinger Liquid is thus broken and this kind of " spin-charge recombination " may be relevant to the generation of well-defined quasi-particles in superconducting state [60]. Under scaling to lower energy, $2gd_u$ is renormalized to divergence, so Θ_c and Φ_s oscillate around stable equilibrium positions and gaps open in both spin and charge excitations, which leads to non-magnetic ground state dominated by singlet superconducting fluctuations. For clarity, let's discuss the special case of $K_c = 1$ and $u_s = u_c$. Then H_{1D} can be decoupled into 2 independent Sine-Gordon models of $\Phi_\pm = \frac{1}{\sqrt{2}}(\Theta_c \pm \Phi_s)$, corresponding to 2 branches of free massive fermions. In this case, both spin gap and charge gap are equal, that is

$$\Delta_c = \Delta_s \propto 2\pi|g|d_u \propto \frac{V^2}{J\Delta_0}.$$

In general, the effect of small $|K_C - 1| > 0$ is only to mix the above two branches together, while the qualitative picture of gap formation remains robust. Further more, at leading order , it is expected that $\Delta_{s,c} \propto \frac{V^2}{J\Delta_0} \propto |g_1|^{1/2}$ is a fairly good approximation to start with [60], . One can associate this gap with the superconducting gap Δ_{sc}, identified as the quasiparticle gap measured for example by ARPES in superconducting state.

Now let's discuss the experimental consequences of the two quantitatively different energy scales Δ_n and Δ_{sc} derived above.

1. overdoping limit of the gaps: In going toward overdoped region, because RVB correlation is significantly suppressed, the relevant energy scale g is reduced to $J\Delta_0$, instead of $V^2/J\Delta_0$ [60]. Because $\frac{\Delta_n}{\Delta_{sc}} \propto \exp\left(\frac{-v}{\pi a g^2}\right)$, Δ_n is much more suppressed compared with Δ_{sc} [60], which is well consistent with the ARPES results [56], and extensive experimental evidences supporting the "absence" of normal state gap in overdoped region.

2. Single partical spectrum: by combining the present scenario with the spectral properties of Luther-Emery system [61] , one can understand the broad "edge" feature near (π,0) in ARPES of underdoped normal state, as due to the proximity toward charge gap formation that turns the power law singularity ($\propto \omega^{\alpha-1/2}$,$0 < \alpha << 1/2$) into a non-singular edge in $A(k,\omega) \propto \omega^{\alpha-1/2}$ ($\alpha > 1/2$) [60]. However, this singularity is restored in overdoped region where the effect of RVB background on stripe is much weakened, therefore singular peaks with long tails are preserved in $A(k,\omega)$ spectrum, as is consistent with what was observed in ARPES [62] . 3. Tc of Superconductivity : In superconducting state, global phase coherence allows single particle hopping between adjacent stripes

through higher order process. From the calculation of corresponding matrix element $t' \approx \frac{\hbar^2}{2m^* d^2}$($d$ is inter-stripe distance) , one can extract the effective mass $1/m^* \propto \frac{V^2}{J\Delta_0}$ (underdoped case), and thus estimate the Josephson coupling energy $E_J \approx \frac{\hbar^2 \rho_s}{2m^* d} \propto \frac{\Delta_{sc}}{d}$, where ρ_s is the superfluid density of a single stripe [60] . In underdoped region, one can attribute superconducting transition to the global phase ordering [49] and therefore $T_c \propto E_J \propto x\Delta_{sc}$, which agrees well with two facts: first, $T_c \propto x$; second, $T_{c,max}$ scales with Δ_{sc} among the cuprates family. In overdoped region, $T_c \propto \Delta_{sc} \propto J\Delta_0$ because d is saturated and a new energy scale $J\Delta_0$ takes the place of $\frac{V^2}{J\Delta_0}$, this is consistent with the BCS like relation observed in overdoped cuprates .

4. "heavy mass" issue: We can treat the "heavy mass" issue raised recently in [63] within the present picture: in underdoped cuprates, $\frac{k_B T_c}{x} = \hbar v^* \propto \Delta_{sc} \propto V^2/J\Delta_0$ and is roughly doping-independent. It can be connected to the flat dispersion perpendicular to horizontal stripes (Γ to (0,π) direction), as suggested in [63], and can be attributed to slow hole motion transverse to stripes [60], which limits the achievement of higher T_c.

Before concluding this section, the significance of the above scenario is commented as follows: First, it opens new route toward the understanding of the subtle relation between pseudo-gap and superconducting gap, in that both Δ_n and Δ_{sc} have the same origin : strong pairing interaction in RVB background, but can be quantitatively different in their dependences on V and $J\Delta_0$. Secondly, one can unify the important energy scales : Δ_n, Δ_{sc}, E_J, T_c, by determining their unique dependences on a single parameter ($V^2/J\Delta_0$ in underdoped region and $J\Delta_0$ in overdoped region), this explains the material-independent scaling in Δ_{sc} : Δ_n : $T_{c,max}$ among cuprates family, while a single material-independent J can not.

In summary, I model the stripe phase in high T_c cuprates as a single stripe coupled to the RVB spin liquid background by the single particle hopping. In normal state, the strong pairing interaction inherent in RVB state is therefore transfered into the Luttinger stripe and drives it toward Luther-Emery Stripe with spin-gap formation. The establishment of global coherence in superconducting state contributes to a more relevant coupling to the stripe and leads to gap opening in both spin and charge sectors.

After understanding "what RVB can do for stripes", it is natural to ask the questions in the other direction, namely "what stripes can do to affect RVB background". This issue will be addressed in the following section.

5. HOW STRIPE DYNAMICS AFFECTS RVB SPIN LIQUID

In the previous section we have explored a scenario [64] where stripes are coupled to an RVB (Resonating Valence Bond) spin liquid background through single-particle hopping, which results in the generation of two quantitatively different gaps (normal state pseudo-gap and superconducting gap) by strong pairing correlation inherent in the RVB environment. In that scenarios, the dynamical stripes play the central role in accommodating the seeds of Copper pairs for the later establishment of superconductivity order which results from the overlaps of the SC wave function between neighboring one dimensional superconducting stripes, while the hole deficient regions between stripes are less relevant and treated as being remnant of the undoped antiferromagnet with reduced local magnetization which could only compete with the superconducting order. It is then natural to ask: Is it the only possible route toward high T_c superconductivity in striped phase of cuprates? In this section, we will attempt to explore an alternative scenario based on the same microscopic model used in the previous section, albeit from a different viewpoint of the role of stripe dynamics, namely, the effect of dynamical stripes in turning a pre-paired RVB spin liquid into a superconductor [65].

Histrocally the relevance of RVB spin liquid to high T_c superconductivity was suggested by Anderson more than a decade ago [1]. The basic idea is that the undoped cuprates have a novel quantum disordered ground state which is the resonating superposition of different configurations of local singlet pairs (so called valence bonds). Upon hole doping, these localized electron pairs are gradually liberated and become Cooper pairs which condense into a superconducting ground state. Though it remains inconclusive after more than a decade [66], We note that beyond the detailed formulation of RVB theory, there is one important aspect which has enjoyed a broader acceptance , that is, strong local pairing correlation is present even in the normal state of under-doped cuprates although the global phase coherence is established only at a lower temperature. Despite the general argument by Emery and Kivelson [67] about the effect of low carrier density on reducing phase stiffness, the concrete mechanism that is responsible for turning a pre-paired but incoherent RVB liquid into a superconductor with global phase coherence, is albeit unclear. All these motivate us to explore this issue in the context of dynamic stripes and its impact on phase ordering in the RVB spin liquid which might gives rise to a global superconducting order.

5.1 Formulation

Based on the two-component stripe picture suggested in ref [64], one can start with the total microscopic Hamiltonian as follows,

$$\begin{aligned} H(c,c^{+},d,d^{+}) &= H_{1D}(d,d^{+}) + H_{RVB}(c,c^{+}) \\ &+ H_{couple}(c,c^{+},d,d^{+}), \end{aligned} \tag{11}$$

where c, c^{+} and d, d^{+} represent the annihilation and creation operators of a single electron in 2D RVB background and 1D stripe, respectively. In the undoped background, there is on average one electron per site and the charge degree of freedom is frozen (which is however gradually mobilized when the coupling with stripes develops with hole doping), and only spin exchange interaction is relevant at low energy scale, which is responsible for the singlet formation in RVB state. Within the stripes, where doped holes concentrate, both charge and spin degrees of freedom are active at low energy scale.

Now, let us treat these three parts one by one as follows in order to obtain the low energy effective theory:

Hamiltonian of 2D RVB spin liquid H_{RVB}: We start with the 2D antiferromagnetic Heisenberg model and perform a routine Hartree-Fork decoupling which leads to the RVB Hamiltonian H_{RVB} [1],

$$\begin{aligned} H_{RVB} &= J\sum_{<ij>}(S_iS_j - 1/4) \\ &= -J\sum_{<ij>} b_{ij}^{+}b_{ij} \\ &= -J\sum_{<ij>}(\Delta_{ij}b_{ij}^{+} + h.c. - |\Delta_{ij}|^2), \end{aligned} \tag{12}$$

where $b_{ij}^{+} = \frac{1}{\sqrt{2}}[c_{i,\uparrow}^{+}c_{j,\downarrow}^{+} - c_{i,\downarrow}^{+}c_{j,\uparrow}^{+}]$. Integrating over fermion variables, one can get the "free energy " of RVB order parameters(OP) Δ_{ij}:

$$\begin{aligned} F_{RVB} &\approx a\sum_{<ij>}|\Delta_{ij}|^2 + b\sum_{<ij>}|\Delta_{ij}|^4 \\ &+ c\sum_{plaquette[ijkl]}(\Delta_{ij}^{*}\Delta_{jk}\Delta_{kl}^{*}\Delta_{li} + h.c.) + ..., \end{aligned} \tag{13}$$

where a,b,c are parameters derived from microscopic model calculations. Then approaching the continuous limit by coarse graining:

$$\Psi(\vec{r} \leftarrow \frac{\vec{r}_i + \vec{r}_j}{2}) \leftarrow \quad local \quad average \quad of \quad |\Delta_{ij}|\exp(i\theta_{ij}),$$

one can arrive at the following effective action

$$S_{RVB}^{eff} = \int d\tau dxdy[a'|\Psi|^2 + b'|\Psi|^4 - c'|\Psi|^2|\nabla\Psi|^2], \tag{14}$$

where $a' < 0$, $b' > 0$, $c' < 0$ are renormalized parameters from a,b,c by coarse graining [68]. Note that the temporal phase stiffness ($\propto \frac{1}{U}$, where U is the local charging energy) is rather weak compared with the spatial phase stiffness $E_s \propto |\Psi|^2$ and thus does not appear in Eq[14]. This is because charge fluctuation is significantly suppressed by strongly repulsive interactions, which leads to severe phase fluctuations because of the conjugating relation between phase variable and pair density. This accounts for the absence of phase coherence in half-filled RVB spin liquid. We note that the lack of phase coherence (or the freezing of charge fluctuations) is what distinguishes RVB spin liquid from a superconductor of Cooper pairs.

Hamiltonian of 1D stripes H_{1D}: As shown by extensive experiments, stripe dynamics includes the longitudinal charge and spin fluctuations along the stripe and the transverse motion which is relatively slow. Therefore one can treat the stripes as 1D electron gas(1DEG) at first and then take the transverse degree of freedom into account by doing the suitable average over the transverse configurations. The theory of Luttinger Liquid provides a powerful tool for the description of 1DEG. Following the notation of Luttinger Liquid [59], the low energy effective theory of 1D stripes is described by separated charge modes and spin modes as follows [69]:

$$\begin{aligned} H_{1D}^{eff} &= \int dx[\frac{K_c u_c}{2}\Pi_c^2 + \frac{u_c}{2K_c}(\partial_x\Phi_c)^2] \\ &+ \int dx[\frac{u_s}{2}\Pi_s^2 + \frac{u_s}{2}(\partial_x\Phi_s)^2 + g_1\cos(\sqrt{8\pi}\Phi_s)], \end{aligned} \tag{15}$$

where Φ_c, Π_c, and Φ_s, Π_s , are conjugated boson operators representing density fluctuations in charge and spin sectors of 1D Luttinger Liquid, respectively. u_c, u_s are the corresponding propagating velocities, and K_c is a parameter of interaction. The last term $g_1\cos(\sqrt{8\pi}\Phi_s)$ is the spin gap correction caused by the coupling between 1D stripes and 2D RVB background with strong pairing correlations (see ref [64] for detail).

Coupling between 1D stripes and 2D RVB background H_{couple}:

$H_{couple}(c, c^+, d, d^+) = \sum_{k,q,\sigma} V c^+_{k,\sigma} d_{q,\sigma}\delta_{k_x,q} + h.c.$, where only horizontal stripe (along x direction) is considered, and momentum conservation is ensured by requiring $k_x = q$. The coupling term accounts for the single electron hopping between stripes and the background and V gives the hopping matrix element. In order to discuss the low energy effective theory with boson variables only, we need to integrate out fermion variables so the lowest order relevant process happens at the second order of V. For the present discussion, we adopt the Cooper pair tunneling process [48] as the only effective coupling between 2D RVB environment and 1D stripes, that is

$$H_{couple}^{eff}(\Psi,\Delta) = g\int dxdy\Psi(x,y)\Delta^*(x)f(y-Y) + h.c., \tag{16}$$

where g is the continuous limit of pair tunneling amplitude, $\Psi(x, y)$ stands for the coarse grained RVB order parameter , $\Delta(x)$ is the singlet pairing order parameter of stripes, $f(y-Y)$ gives the transverse distribution function of the stripe position due to vibration , and Y stands for the transverse displacement of the stripe diffusion. Here we assume that the stripe transverse modes can be described as the superposition of the fast mode of vibration and the slow mode of diffusion. Under the harmonic approximation, one can assume $f(y) \propto \sqrt{\alpha}\exp(-\alpha y^2)$, where $1/\sqrt{\alpha}$ represents the amplitude of stripe vibration mode which is determined by the microscopic details which are responsible for the stripe formation and stabilization. In Nd doped LSCO where stripes are pinned by the Low Temperature Tetragonal lattice structure [28,30,39,70,71], $1/\sqrt{\alpha}$ is expected to be small (of the order of one lattice unit); while in optimum doped LSCO and YBCO, stripes are more disordered and transverse fluctuations are strong, so $1/\sqrt{\alpha}$ can be as large as the order of inter-stripe distance which is about $4a$ (a is the lattice unit).

In the formulation of Luttinger Liquid [59],

$$\Delta(x) = d_u \exp(-i\sqrt{2\pi}\Theta_c) \sin(\sqrt{2\pi}\Phi_s).$$

We note that this pair tunneling process is very important in the present picture. Through this process the previously localized spin singlets in RVB background are mobilized and the charge degree of freedom is resumed, therefore one can no longer distinguish spin singlets of electrons from Cooper pairs which constitute the basis of superconductivity, and the RVB phase variable can be continuously connected to the phase degree of freedom of d-wave superconductivity order parameter.

After obtaining the effective Hamiltonians of the coupled RVB and stripe variables, we can study the effect of stripe dynamics on the RVB spin liquid background (the phase degree of freedom in particular) by integrating out the stripe variables (including both the OP field $\Delta(x)$ and the transverse mode variable Y)

$$\begin{aligned} \exp[-\Delta S_{RVB}^{eff}] &= \int D\Phi_c(x,\tau) D\Phi_s(x,\tau) DY(\tau,x) \\ & \exp[-\int_0^\beta d\tau (H_{1D}^{eff} + H_{couple}^{eff} + H_{tm})], \end{aligned} \tag{17}$$

where $H_{tm}$is the Hamiltonian of the transverse modes of stripes, $\beta = \frac{1}{k_B T}$. Therefore, the integration up to the second order gives

$$\begin{aligned} \Delta S_{RVB}^{eff} &\approx -\frac{g^2}{2} \int d\tau d\tau' dx dy dx' dy' \Psi(x,y,\tau) \Psi^*(x',y',\tau') \\ & \langle \Delta^*(x,\tau)\Delta(x',\tau')\rangle_{1D} \langle f(y - Y(\tau,x)) f(y' - Y(\tau',x'))\rangle_{tm} \end{aligned} \tag{18}$$

where $\langle \ \ \rangle_{1D}$ and $\langle \ \ \rangle_{tm}$ stand for average over longitudinal and transverse stripe variables, respectively. According to the Luttinger Liquid theory,

$$\langle \Delta^*(x,\tau)\Delta(x',\tau')\rangle_{1D} \approx \begin{cases} \frac{d_u^2}{|(\tau-\tau')+i\frac{(x-x')}{u_c}|^{\frac{1}{K_c}} |(\tau-\tau')+i\frac{(x-x')}{u_s}|} & |(\tau-\tau')+i\frac{(x-x')}{u_{s,c}}| << \frac{\xi_s}{u_{s,c}} \\ \frac{d_u^2}{|(\tau-\tau')+i\frac{(x-x')}{u_c}|^{\frac{1}{K_c}} \xi_s/u_s} & |(\tau-\tau')+i\frac{(x-x')}{u_{s,c}}| >> \frac{\xi_s}{u_{s,c}} \end{cases}$$

where $\xi_s \propto 1/\Delta_s$ is the cutoff of length scale given by the spin gap $\Delta_s \propto \sqrt{|g_1|} \exp(\frac{v}{2\pi g_1})$ [64].

In order to extract the spatial and temporal phase stiffness coefficients, one can expand the integrand with respect to $\Delta x = x' - x$, $\Delta y = y' - y$, $\Delta\tau = \tau' - \tau$, over which one can perform integrations [72], then reach

$$\Delta S_{RVB}^{eff} \approx \int d\tau dx dy [E_\tau |\frac{\partial \Psi}{\partial \tau}|^2 + \Delta E_x |\frac{\partial \Psi}{\partial x}|^2 + \Delta E_y |\frac{\partial \Psi}{\partial y}|^2],$$

where the induced incipient temporal phase stiffness is

$$
\begin{aligned}
E_\tau \;&\propto\; \delta g^2\alpha\int_0^{\frac{\xi_s}{u_{c,s}}} d\Delta\tau\langle\exp\left[-\alpha(Y(\tau+\Delta\tau,x)-Y(\tau,x))^2/2\right]\rangle_{tm} \\
&(\Delta\tau)^{1-1/K_c}+\delta g^2\alpha\int_{\frac{\xi_s}{u_{c,s}}}^{\infty} d\Delta\tau\langle\exp\left[-\alpha(Y(\tau+\Delta\tau,x)-Y(\tau,x))^2/2\right]\rangle_{tm} \\
&(\Delta\tau)^{2-1/K_c},
\end{aligned}
\tag{19}
$$

Considering that $\langle(Y(\tau+\Delta\tau,x)-Y(\tau,x))^2\rangle_{tm}=2D(\Delta\tau)$ (where the stripe diffusion is modeled as random walk and D is the diffusive coefficient), then the above expression can be simplified to

$$
E_\tau \propto \delta g^2\alpha(\alpha D)^{1/K_c-2}F(\frac{\alpha D\xi_s}{u_c})+\frac{u_{c,s}}{\xi_s}\delta g^2\alpha(\alpha D)^{1/K_c-3}G(\frac{\alpha D\xi_s}{u_c}),
\tag{20}
$$

where $F(X)=\int_0^X x^{1-1/K_c}e^{-x}dx$, $G(X)=\int_X^\infty x^{2-1/K_c}e^{-x}dx$. At the limit $\frac{\alpha D\xi_s}{u_c}>>1$, $E_\tau\propto\delta g^2\alpha(\alpha D)^{1/K_c-2}$; while for $\frac{\alpha D\xi_s}{u_c}<<1$, $E_\tau\propto\delta g^2\alpha(\alpha D)^{1/K_c-3}/\xi_s$.

The corrections to spatial phase stiffness ΔE_x and ΔE_y can be calculated similarly, however, compared with the unperturbed E_s of 2D RVB effective theory in eq[14] they are negligibly small when δ is small enough.

Now combined with eq[14] where the spatial phase stiffness $E_s\propto\rho^2$ is given, and retain only phase variables (assuming frozen amplitude $|\Psi|=\rho$), one can discuss the phase ordering process in the RVB liquid with the following effective action:

$$
\begin{aligned}
S_{eff} \;&=\; \int_0^\beta d\tau[\sum_i E_\tau\rho^2|\frac{\partial\phi(\vec{r}_i)}{\partial\tau}|^2 \\
&-\; E_s\rho^2\sum_{<ij>}\cos(\phi(\vec{r}_i)-\phi(\vec{r}_j))].
\end{aligned}
\tag{21}
$$

Notice here we turn from the Ginzburg-Landau like "soft-spin" effective model into "hard spin" XY model, because in (2+1) dimension they belong to the same universal class and thus have the same critical behavior.

This effective action has been under heavy discussions in the study of granular superconducting film and Josephson junction array [73]. To reveal the quantum critical physics inside it, one can perform a standard Hubbard-Stratonovich transformation to decouple the Josephson term [74], which introduces the complex order-parameter field ψ in proportion to the expectation value of $\exp(i\phi)$. The resulting Ginzburg-Laudau action in (2+1)D reads:

$$
\begin{aligned}
F_{eff}[\psi] \;&=\; \int dxdyd\tau[\frac{1}{8E_s\rho^2}|\nabla\psi|^2+128E_\tau^3\rho^6|\frac{\partial\psi}{\partial\tau}|^2 \\
&+\; (\frac{1}{2E_s\rho^2}-4E_\tau\rho^2)|\psi|^2+\kappa|\psi|^4].
\end{aligned}
\tag{22}
$$

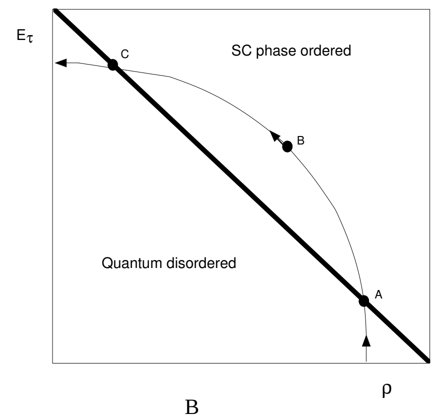

Figure 2: (a).The zero temperature phase diagram of the effective action describing RVB p hase variable coupled to stripe dynamics. The thick line marks the phase boundar y between the SC ordered and disordered phases, while the thin curves represent contours with equal T_{cr}'s. (b).The route followed by a cuprate under hole doping in the ground state phase diagram. $A \to B$ and $B \to C$ correspond to under-doped and over-doped regions, respectively. Both axes use logarithmic scale with arbitrary unit.

So the quantum critical point(QCP) is given by $\frac{1}{2E_s\rho^2} - 4E_\tau\rho^2 = 0$, which separates the zero-temperature phase diagram into superconducting ordered phase ($E_\tau E_s\rho^4 > 1/8$) and non-superconducting disordered phase ($E_\tau E_s\rho^4 < 1/8$). At finite temperature, there exists a crossover temperature $T_{cr}(\rho, E_\tau) \propto \sqrt{\frac{|E_\tau\rho^2 - \frac{1}{8E_s\rho^2}|}{E_\tau^3\rho^6}}$, above which lies the quantum critical region where physical quantities obey scaling laws with T. On the SC ordered side the crossover temperature becomes the transition temperature corresponding to the well-known KT transition [75]. The phase diagram is shown in Fig.2(a). Note that strong asymmetry exists in T_{cr} around the QCP, and the much higher crossover temperature on the disordered side compared with the SC ordered side can explain why the anomalous T dependent scaling behaviors are prevalent in the normal states of superconducting cuprates while in slightly doped insulating cuprates the critical regime eludes experiments (it is however likely that stripe ordering itself can lead to critical scaling behavior which is not considered here).

5.2 Discussions

Now let us discuss how to relate this QCP with the general phase diagram of high T_c cuprates. One can see the quantum phase transition is tuned by a single parameter $E_\tau E_s\rho^4 \propto \delta g^2\rho^6 H(\xi_s, \alpha, D)$ which is a complicated function of doping density δ, RVB OP amplitude ρ, spin gap $1/\xi_s$ and stripe transverse mode parameters α and D. In real world experiments on cuprates, upon hole doping, all the other parameters change accordingly. For example, RVB OP amplitude ρ and spin gap both decrease with doping (experimentally spin gap closes around $\delta = 0.2$), while stripe transverse modes may depend on material-dependent properties like lattice distortions and impurity effects. Therefore a comprehensive understanding of this issue is beyond the scope of this paper. Here to give a qualitative demonstration of the physical mechanism , I will attempt to consider some of the relevant parameters (while leave the others like those of stripe transverse modes as external inputs) and mark the route which might be followed by a cuprate under hole doping in the ground state phase diagram (Fig.2(b)).At first, with slightly doping from the parent cuprate, E_τ increases from zero (roughly in proportion to δ) while ρ gradually decreases from the maximum, therefore at a critical doping value ($\delta_{c1} \approx 0.06$) the system crosses the phase boundary into the SC ordered state. Then upon further doping from under-doped to over-doped regions, δ gradually becomes saturated, meanwhile a diminishing spin gap pushes E_τ toward the limit value controlled by α and D. Therefore the route is bent toward E_τ axis thanks to the decreasing ρ (because over-doping reduces RVB correlations significantly with excessive holes "overflowing" into the background, which is also consistent with the result of RVB mean field calculations [66]). Finally as $\delta > \delta_{c2} \approx 0.3$ the system crosses the phase boundary again and returns to the disordered non-superconducting ground state. During the above process, T_{cr} increases from zero to its maximum and then decreases back to zero, as it is the case for the transition temperature [76].

Then let's address the role of transverse stripe modes in affecting the SC transition. According to eq[20], lower α and D tends to strengthen E_τ (which is especially effective in under doped region where the spin gap is substantial and the limit $\frac{\alpha D \xi_s}{u_c} << 1$ can be approached, assuming $K_c > 1/2$ which coincides with the condition under which SC fluctuations along stripes are relevant at low energy [77]). This suggests that in the present mechanism, larger transverse vibration amplitude ($\approx 1/\sqrt{\alpha}$) favors SC while the diffusion mode does not. Considering the various stripe phases proposed in literature [77], it is interesting to note that SC order is favored only in the intermediate region between the stripe crystal phase (with small vibration amplitude, or large α) and stripe liquid phase (where stripes are meandering strings and diffusions dominate), which implies a very subtle relation between SC order and stripe charge order.

Finally I will briefly compare the present picture with the one suggested in ref [49]: in that work, the superconducting order is induced by the Josephson tunneling between neighboring stripes and it is natural to expect this coupling to be strongly dependent on the inter-stripe distance (presumably decays exponentially with the distance) and also the extent of disorder in stripe configurations, which makes it a subtle issue to explain the simple and well-defined relation between T_c, zero temperature superfluid density and doping density, and the fact that higher T_c is found in the cuprates with more disordered stripe correlations. In the present work, the induced temporal phase stiffness only depends on the local coupling between one stripe and its neighboring background and is therefore not sensitive to the disorder in the coupling between the neighboring stripes.

In conclusion, the low energy effective theory of the RVB phase variable coupled to the stripe dynamics is obtained, where the effect of stripe dynamics induces doping dependent incipient temporal phase stiffness in the RVB liquid, which tunes a quantum phase transition toward a superconducting ground state with global phase order.

6. Summary

To summarize this papaer, we have explored the interplay between RVB spin liquid and 1D stripes in search of novel physics which might be relevant to high Tc cuprates, which is motivated by both experimental evdidence of stripe phase and theoretical interest in studying new state of matter. Indeed, the simple coupling between a single stripe and the RVB spin liquid background by the single particle hopping already gives rise to interesting physics: In the direction from RVB background to stripe, we found the strong pairing interaction inherent in RVB state is transfered into the Luttinger stripe and drives it toward Luther-Emery Stripe with spin-gap formation in normal state, while in superconducting state the establishment of global coherence contributes to a more relevant coupling to the stripe and leads to gap opening in both spin and charge sectors; in the opposite direction, the low energy effective theory of the RVB phase variable is significantly affected by the stripe dynamics which induces doping dependent incipient temporal phase stiffness in the RVB liquid and tunes a quantum phase transition toward a superconducting ground state with global phase order. At this stage, it remains an open issue which of the above scenarios is more relevant to the physics of high Tc cuprate if not neither. The well-known presence of d-wave superconducting order parameter in cuprates seems to favor a 2D electron system based scenario such as the second one we discussed, while the first one as a 1D electron system enjoys theoretical simplicity and soundness.

For future work, it will be interesting to introduce more interactions to generalize the present "single strip e plus 2D background" picture, for example, the direct coupling between adjacent stripes. Also we need to eventually consider Columb interaction and antiferromagnetic exchange which were not explicitly treated in the present picture. With the inclusion of complicated interactions which could lead to multiple competing orders, we would need to make use of new orginization principle to sort out things, for example new symmetry principle [78].

Fifteen years after the great discovery of high Tc cuprates, we are still puzzled by this intriguing master piece created by Mother Nature. However thanks to the tremendous progress in its experimental research, we now know much better about the properties of cuprates than we did 15 years ago, which offers us theorists more clues and stronger constraints to reach a final answer, which is hopefully not far away from us.

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